Exercise 6.1

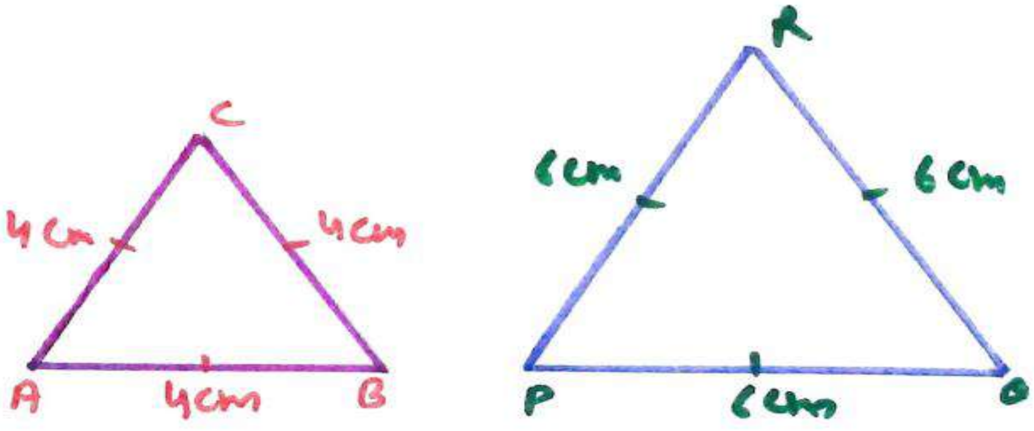
1. Fill in the blanks using the correct word given in brackets:

(i) All circles are …similar…. (Congruent, similar)
(ii) All squares are …similar… (similar, congruent)
(iii) All … equilateral…triangles are similar. (Isosceles, equilateral)
(iv) Two polygons of the same number of sides are similar, if
(a) their corresponding angles are …equal…and
(b) their corresponding sides are ...proportional… (equal, proportional)

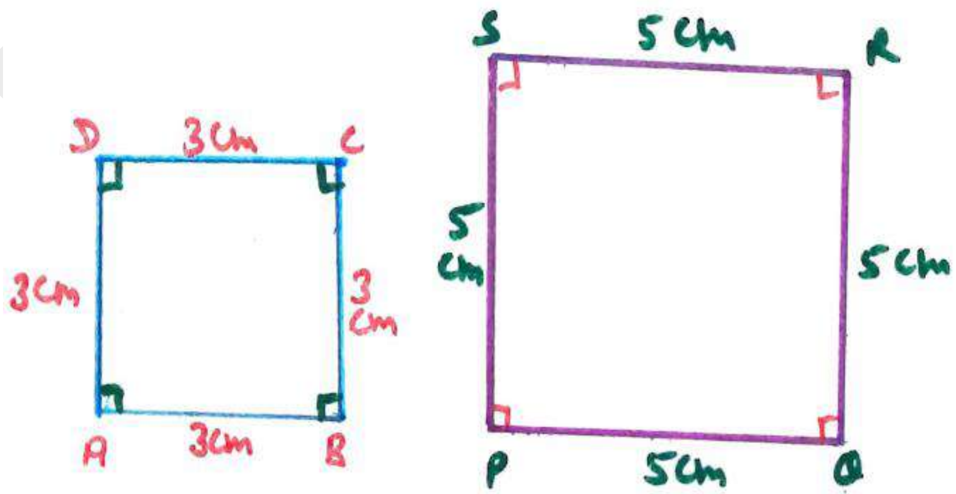
2. Give two different examples of pair of (i) similar figures. (ii) non-similar figures.

Sol. (i) similar figures

1. Two equilateral triangles with sides 4cm and 6cm

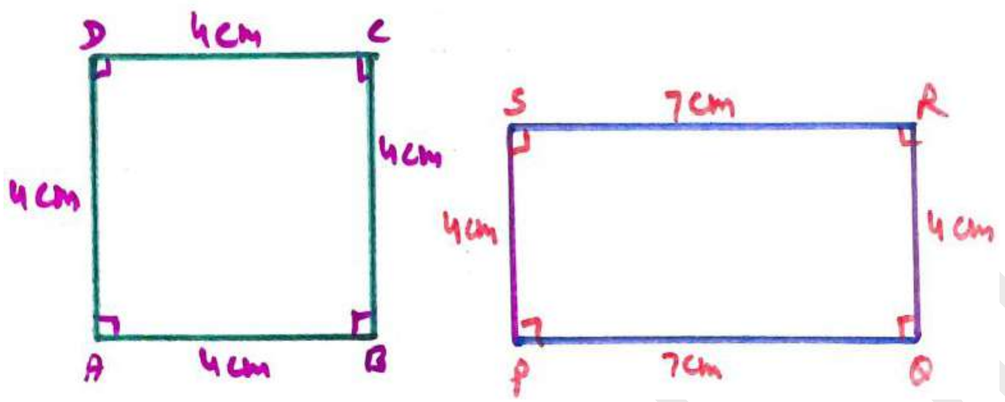


2. two squares with sides 3cm and 5cm

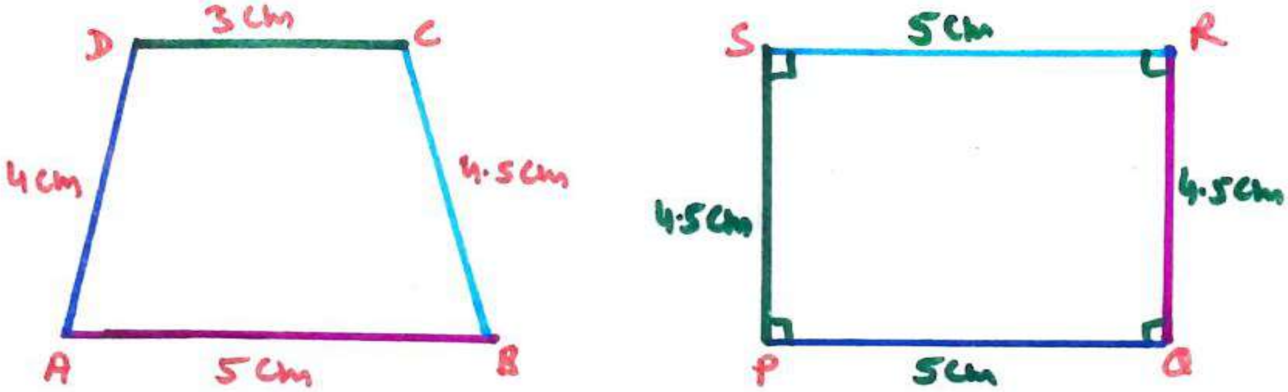


(ii) non-similar figures

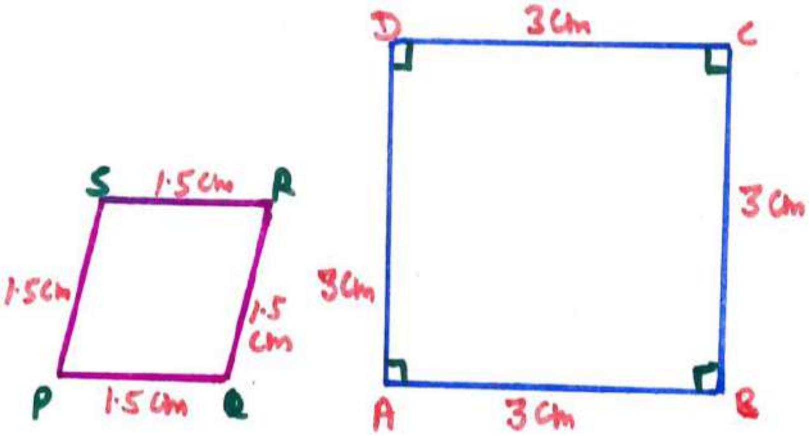
1. Square and rectangle



2. Trapezium and rectangle



3. State whether the following quadrilaterals are similar or not:



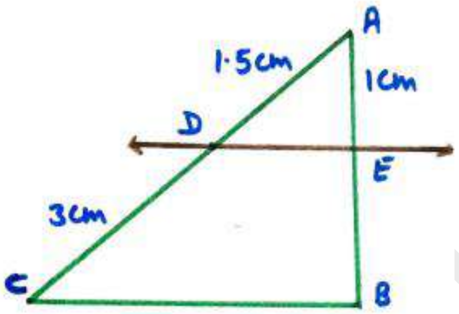
Sol. The given quadrilaterals are not similar.

Exercise: 6.2

1. In Fig. DE || BC. Find EC in (i) and AD in (ii).

Sol. (i) Given: In △ ABC, DE || BC

To find: EC =?



Sol. In △ ABC, DE || BC

$\Rightarrow \frac{\mathrm{AD}}{\mathrm{DB}} = \frac{\mathrm{AC}}{\mathrm{EC}}$ [By B.P. theorem]

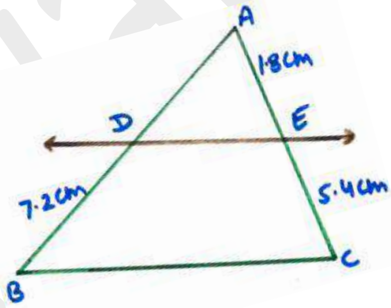
$\Rightarrow \frac{1.5}{3} = \frac{1}{\mathrm{EC}}$

$\Rightarrow \mathrm{EC} = \frac{3\times1}{1.5}$

$\therefore \mathrm{EC} = 2$ cm Ans.

Sol. (ii) Given: In △ ABC, DE || BC

To find: AD =?



Sol. In △ ABC, DE || BC

$\Rightarrow \frac{\mathrm{AD}}{\mathrm{DB}} = \frac{\mathrm{AE}}{\mathrm{EC}}$ [By B.P. theorem]

$\Rightarrow \frac{\mathrm{AD}}{7.2} = \frac{1.8}{5.4}$

$\Rightarrow \mathrm{AD} = \frac{1.8\times7.2}{5.4}$

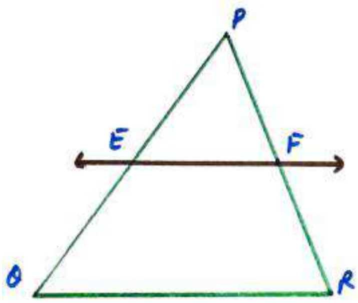
$\therefore \mathrm{AD} = 2.4$ cm Ans.

2. E and F are points on the sides PQ and PR respectively of a △ PQR. For each of the following cases, state whether EF||QR:

(i) $\mathrm{PE} = 3.9$ cm, $\mathrm{EQ} = 3$ cm, $\mathrm{PF} = 3.6$ cm and $\mathrm{FR} = 2.4$ cm

Sol. Given: In △ PQR, $\mathrm{PE} = 3.9$ cm, $\mathrm{EQ} = 3$ cm,
$\mathrm{PF} = 3.6$ cm, $\mathrm{FR} = 2.4$ cm,

To state: whether EF||QR or not.



Proof: Here, $\frac{\mathrm{PE}}{\mathrm{EQ}} = \frac{3.9}{3} = \frac{13}{10}$

And $\frac{\mathrm{PF}}{\mathrm{FR}} = \frac{3.6}{2.4} = \frac{3}{2}$

Here $\frac{\mathrm{PE}}{\mathrm{EQ}} \neq \frac{\mathrm{PF}}{\mathrm{FR}}$

$\therefore$ EF is not parallel to QR.

(ii) $PE = 4 \text{ cm}, QE = 4.5 \text{ cm}, PF = 8 \text{ cm}$ and $RF = 9 \text{ cm}$

Sol. Given: In △ PQR, $PE = 4 \text{ cm}, QE = 4.5 \text{ cm}, PF = 8 \text{ cm}, RF = 9 \text{ cm},$

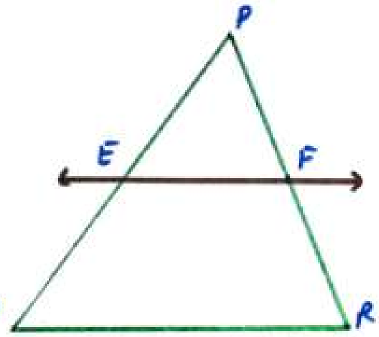
To state: whether EF||QR or not.

Proof: Here, $\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$

and $\frac{PF}{FR} = \frac{8}{9}$

Here, $\frac{PE}{EQ} = \frac{PF}{FR}$

⇒ EF ∥ QR [By the converse of B. P. theorem]



(iii) $PQ = 1.28 \text{ cm}, PR = 2.56 \text{ cm}, PE = 0.18 \text{ cm}$ and $PF = 0.63 \text{ cm}$

Sol. Given: In △ PQR, $PQ = 1.28 \text{ cm}$, $PR = 2.56 \text{ cm}$, $PE = 0.18 \text{ cm}$, $PF = 0.36 \text{ cm}$,

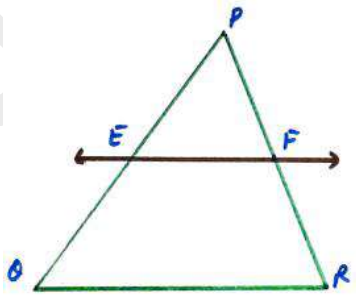
To state: whether EF||QR or not.

Proof: Here, $\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$

And $\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$

Here, $\frac{PE}{PQ} = \frac{PF}{PR}$

⇒ EF ∥ QR [By the converse of B. P. theorem]



3. In Figure, if LM ∥ CB and LN ∥ CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.

Sol. Given: In △ ABC, LM || BC and in △ ADC, LN || CD

To prove: $\frac{AM}{AB} = \frac{AN}{AD}$

Proof: In △ ABC, LM || BC,

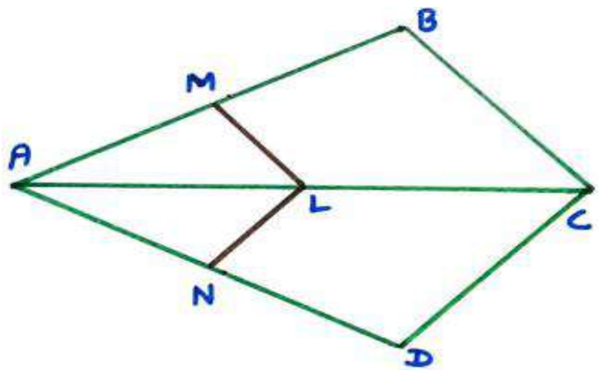
⇒ $\frac{AM}{AB} = \frac{AL}{AC}$ [By B.P. theorem] ......(i)

And in △ ADC, LN || CD,

⇒ $\frac{AM}{AB} = \frac{AN}{AD}$ [By B.P. theorem] .....(ii)

From (i) and (ii), we have

$\frac{AM}{AB} = \frac{AN}{AD}$, proved.



4. In Figure, DE $\parallel$ AC and DF $\parallel$ AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.

Sol. Given: In $\triangle$ ABC, DE $\parallel$ AC and DF $\parallel$ AE

To prove: $\frac{BF}{FE} = \frac{BE}{EC}$.

Proof: In $\triangle$ ABC, DE || AC

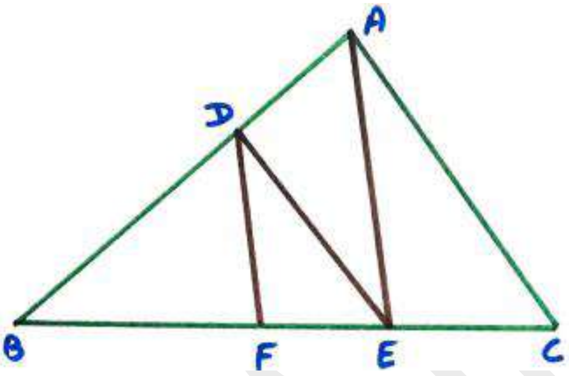
$\Rightarrow \frac{BD}{DA} = \frac{BE}{EC}$ [By B.P. theorem] …………(i)

And in $\triangle$ ABC, DE || AC

$\Rightarrow \frac{BD}{DA} = \frac{BF}{FE}$ [By B.P. theorem] ………(ii)

From (i) and (ii), we have

$\frac{BF}{FE} = \frac{BE}{EC}$, proved.



5. In Figure, DE $\parallel$ OQ and DF $\parallel$ OR. Show that EF $\parallel$ QR.

Sol. Given: In $\triangle$ ABC, DE $\parallel$ OQ and DF $\parallel$ OR

To prove: EF $\parallel$ QR.

Proof: In $\triangle$ POQ, DE || OQ

$\Rightarrow \frac{PE}{EQ} = \frac{PD}{DO}$ [By B. P. theorem] …………(i)

And in $\triangle$ POR, DF || OR

$\Rightarrow \frac{PF}{FR} = \frac{PD}{DO}$ [By B. P. theorem] ……………(ii)

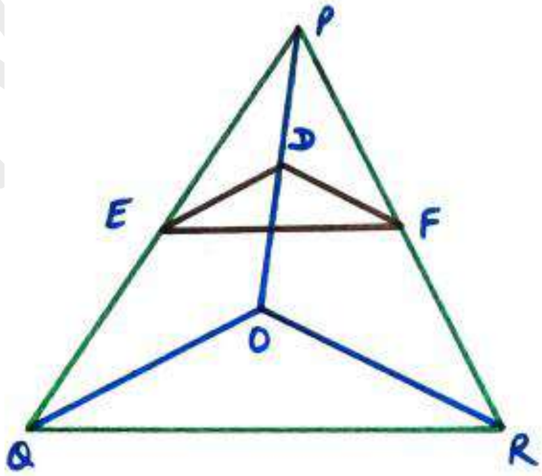
From (i) and (ii), we have

$\frac{PE}{EQ} = \frac{PF}{FR}$

Now, in triangle PQR,

$\frac{PE}{EQ} = \frac{PF}{FR}$

$\Rightarrow$ EF $\parallel$ QR [By the converse of B. P. theorem]



6. In Fig. A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

Sol. Given: In $\triangle$ PQR, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR

To prove: BC || QR.

Proof: In △ POQ, AB || PQ,

$\Rightarrow \frac{OA}{AP} = \frac{OB}{BQ}$ [By B.P. theorem] …………(i)

And in △ POR, AC || PR,

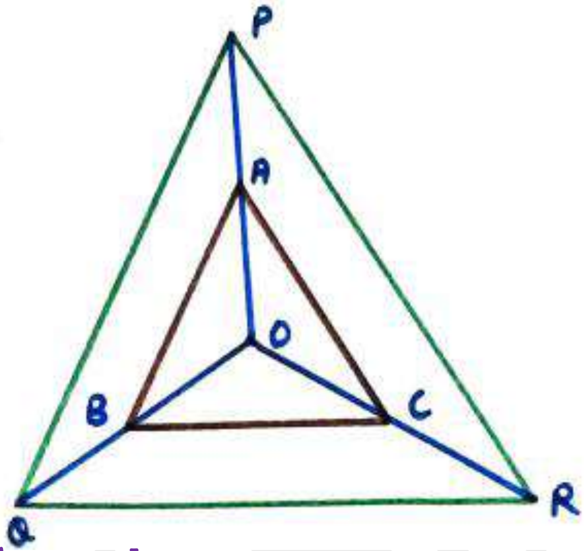
$\Rightarrow \frac{OA}{AP} = \frac{OC}{CR}$ [By B.P. theorem] …………(ii)

From (i) and (ii), we have

$\frac{OB}{BQ} = \frac{OC}{CR}$

Now, in triangle OQR,

$\frac{OB}{BQ} = \frac{OC}{CR}$



⇒ BC ∥ QR [By the converse of B. P. theorem]

7. Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

Sol. Given: D is the mid-point of AB and DE || BC intersects AC at E.

To prove: AE = EC

Proof: Let DE is a line through the mid-point of AB, parallel to BC intersects AC at E. i.e., DE || BC,

In △ ABC, DE || BC

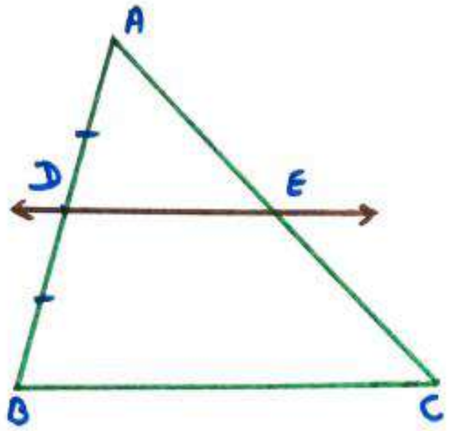
$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$ [By B.P. theorem]

$\Rightarrow \frac{AD}{AD} = \frac{AE}{EC}$ [∵ AD = DB]

$\Rightarrow 1 = \frac{AE}{EC}$

$\Rightarrow AE = EC,$

∴ E is the mid-point of AC.



8. Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side.

Sol. Given: D is the mid-point of AB and DE

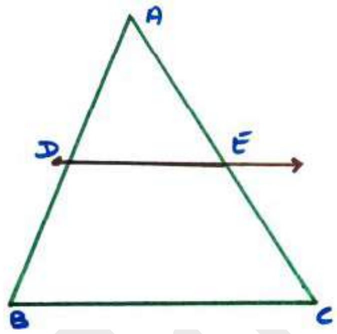
To prove: DE ∥ BC

Proof: Let, DE is a line, which passes through the mid-points of AB and AC.

$\Rightarrow AD = DB$ and $AE = EC$.

$\Rightarrow \frac{AD}{DB} = 1$ and $\frac{AE}{EC} = 1$

or $\frac{AD}{DB} = \frac{AE}{EC}$



Now, in triangle ABC.

$\frac{AD}{DB} = \frac{AE}{EC}$ [Proved above]

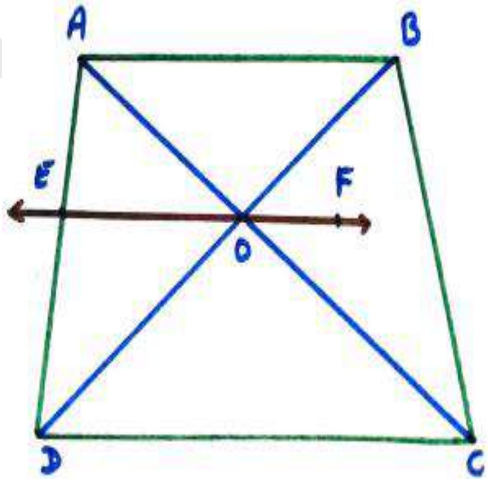
$\Rightarrow$ DE ∥ BC [By the converse of B. P. theorem]

9. ABCD is a trapezium in which AB ∥ DC and its diagonals intersect each at O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Sol. Given: ABCD is a trapezium in which AB ∥ DC and its diagonals intersect each at O.

To prove: $\frac{AO}{BO} = \frac{CO}{DO}$

Construction: Draw a line EF ∥ AB ∥ DC passing through O.



Proof: In △ ABC, EO || AB

$\Rightarrow \frac{AE}{DE} = \frac{BO}{DO}$ [By B.P. theorem] .....(i)

And in △ ADC, EO ∥ CD

$\Rightarrow \frac{AE}{DE} = \frac{AO}{CO}$ [By B.P. theorem] ....(ii)

From the equation (i) and (ii), we have

$\frac{AO}{CO} = \frac{BO}{DO}$

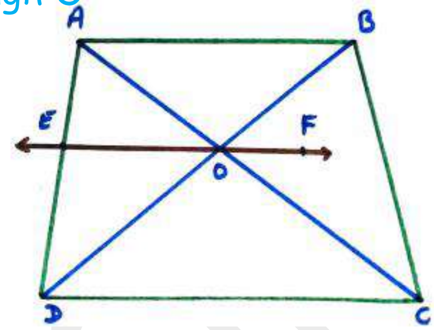
$\Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$ Proved.

10. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Sol. Given: The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$ or $\frac{AO}{CO} = \frac{BO}{DO}$

To prove: $AB \parallel DC$ or $ABCD$ is a trapezium.

Construction: Draw a line EO $\parallel AB$ passing through O



Proof: In $\triangle ABD$, $EO||AB$

$\Rightarrow \frac{AE}{DE} = \frac{BO}{DO}$ [By B.P. theorem]

But, $\frac{AO}{CO} = \frac{BO}{DO}$

$\Rightarrow \frac{AE}{DE} = \frac{AO}{CO}$

$\Rightarrow EO \parallel DC$ [By the converse of B. P. theorem]

But EO $\parallel AB$ [By construction]

$\Rightarrow AB \parallel CD$

$\therefore ABCD$ is a trapezium. Proved.

Exercise: 6.3

1. State which pairs of triangles in Fig. 6.34 are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

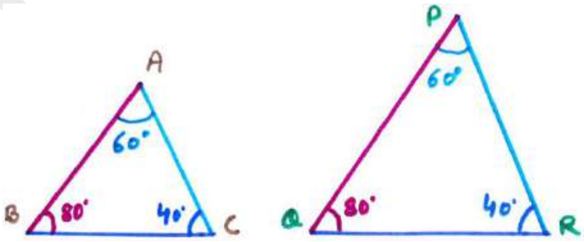
Sol. (i) In $\triangle ABC$ and $\triangle PQR$

$\angle A = \angle P$ [$\because$ each 60°]

$\angle B = \angle Q$ [$\because$ each 80°]

$\angle C = \angle R$ [$\because$ each 40°]

$\therefore \triangle ABC \sim \triangle PQR$ [AAA similarity]



(ii) In $\triangle ABC$ and $\triangle QRP$

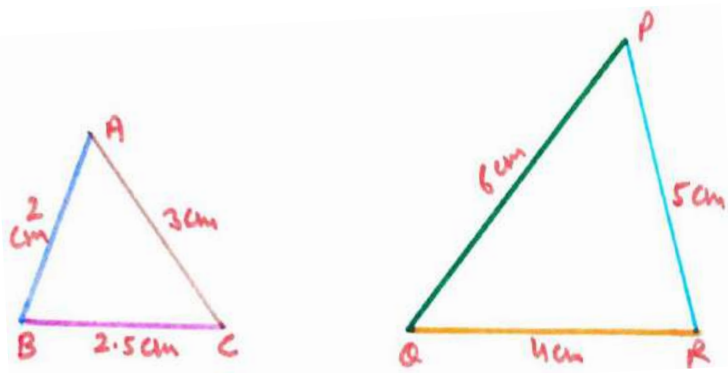
$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}$

$\frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2}$,

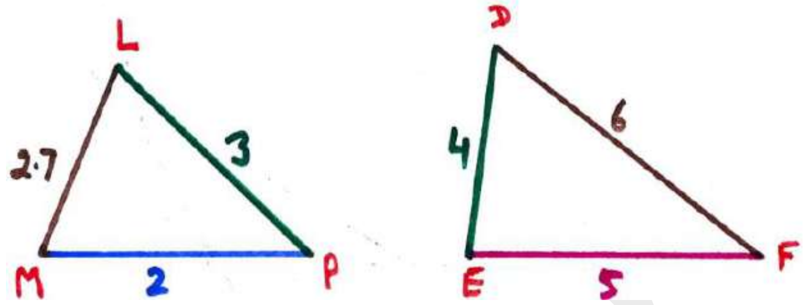
And $\frac{CA}{PQ} = \frac{3}{6} = \frac{1}{2}$

Here $\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ} = \frac{1}{2}$ each

$\therefore \triangle ABC \sim \triangle QRP$ [SSS similarity]



(iii) Triangles are not similar because the corresponding sides are not proportional.



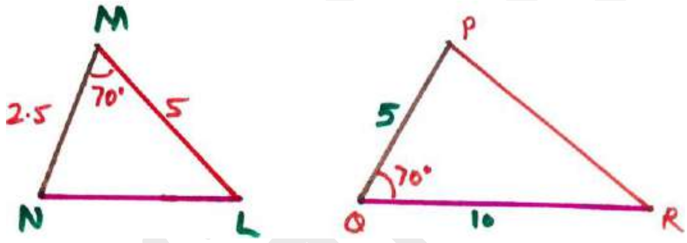
(iv) In $\triangle$ NML and $\triangle$ PQR

Here $\frac{\mathrm{NM}}{\mathrm{PQ}} = \frac{\mathrm{ML}}{\mathrm{QR}} = \frac{1}{2}$ each

$\angle \mathrm{M} = \angle \mathrm{Q}$ [$\because$ each $70°$]

$\therefore$ $\triangle$ NML and $\triangle$ PQR

[SAS similarity]

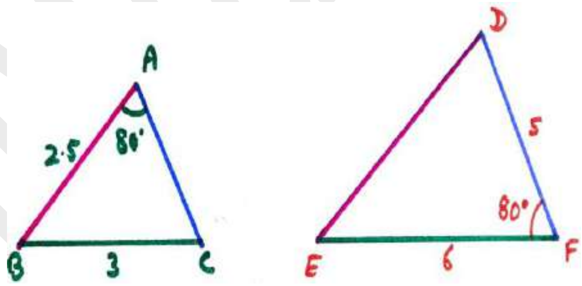


(v) In $\triangle$ ABC and $\triangle$ DFE

Here $\frac{\mathrm{AB}}{\mathrm{DF}} = \frac{\mathrm{BC}}{\mathrm{FD}} = \frac{1}{2}$ each

But $\angle \mathrm{B} \neq \angle \mathrm{F}$ [included angles between the proportional sides]

$\therefore$ Triangles are not similar.



(vi) In $\triangle$ DEF,

$\angle \mathrm{D} + \angle \mathrm{E} + \angle \mathrm{F} = 180°$

$\Rightarrow 70° + 80° + \angle \mathrm{F} = 180$

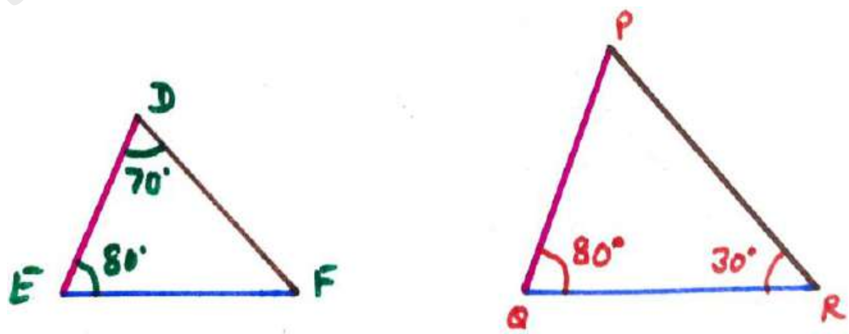
$\Rightarrow \angle \mathrm{F} = 30°$

Similarly, in $\triangle$ PQR,

$\angle \mathrm{P} + \angle \mathrm{Q} + \angle \mathrm{R} = 180°$

$\Rightarrow \angle \mathrm{P} + 80° + 30° = 180°$

$\Rightarrow \angle \mathrm{P} = 70°$



In $\triangle$ DEF and $\triangle$ PQR,

$\angle \mathrm{D} = \angle \mathrm{P}$ [$\because$ each $70°$]

$\angle \mathrm{E} = \angle \mathrm{Q}$ [$\because$ each $80°$]

$\angle \mathrm{F} = \angle \mathrm{R}$ [$\because$ each $30°$]

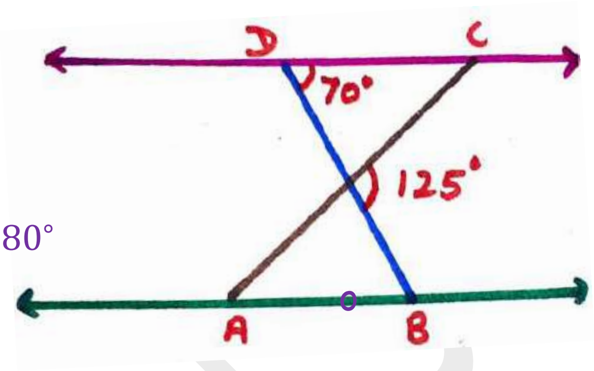
$\therefore \triangle \mathrm{DEF} \sim \triangle \mathrm{PQR}$ [AAA similarity]

2. In Figure, $\triangle ODC \sim \triangle OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.

Sol. Given: $\triangle ODC \sim \triangle OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$

To Find: $\angle DOC$, $\angle DCO$ and $\angle OAB$.

Sol. Since DOB is a straight line.



$\therefore \angle DOC + \angle COB = 180^\circ$

$\Rightarrow \angle DOC = 180^\circ - 125^\circ$

$= 55^\circ$

In $\triangle DOC$, $\angle DCO + \angle CDO + \angle DOC = 180^\circ$

$\Rightarrow \angle DCO + 70^\circ + 55^\circ = 180^\circ$

$\Rightarrow \angle DCO + 125^\circ = 180^\circ$

$\Rightarrow \angle DCO = 180^\circ - 125^\circ$

$\Rightarrow \angle DCO = 55^\circ$

Given that, $\triangle ODC \sim \triangle OBA$.

$\therefore \angle OAB = \angle OCD$ [Corresponding angles of similar triangles]

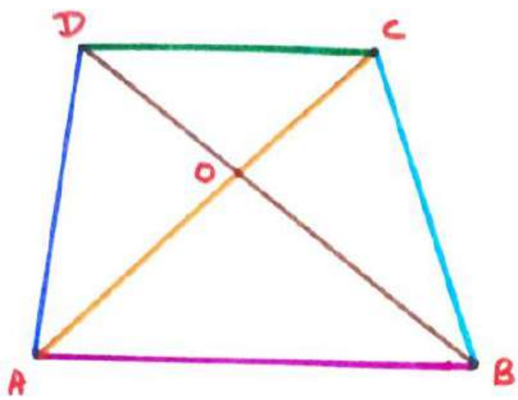
$\Rightarrow \angle OAB = 55^\circ$

$\therefore \angle DOC = 55^\circ$, $\angle DCO = 55^\circ$ and $\angle OAB = 55^\circ$ Ans.

3. Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Sol. Given: In trapezium ABCD, $AB \parallel DC$ and diagonals AC and BD intersect each other at the point O.

To prove: $\frac{OA}{OC} = \frac{OB}{OD}$



Proof: In $\triangle BOA$ and $\triangle DOC$

$\angle BOA = \angle DOC$ [Vertically opposites angles]

$\angle ABO = \angle CDO$ [Alternate interior angles]

$\angle BAO = \angle DCO$ [Alternate interior angles]

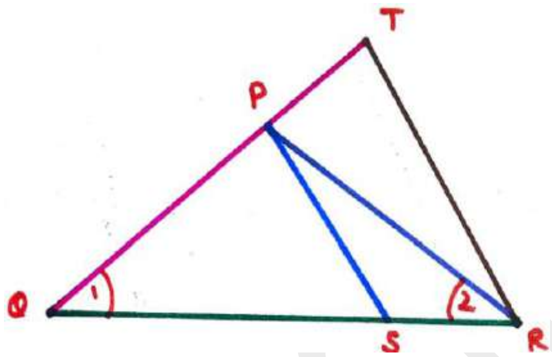
$\therefore \triangle BOA \sim \triangle DOC$

$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$

[$\because$ Corresponding parts of similar triangles are proportional]

4. In figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\triangle PQS \sim \triangle TQR$.

Sol. Given: $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$



To prove: $\triangle PQS \sim \triangle TQR$

Proof: In $\triangle PQR$,

$\angle PQR = \angle PRQ$

$\therefore PQ = PR \ldots (1)$

Given that,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

$\Rightarrow \frac{QR}{QS} = \frac{QT}{QP}$ [From (i) $PQ = PR$] $\ldots\ldots (2)$

$\therefore$ In $\triangle$ PQS and $\triangle$ TQR

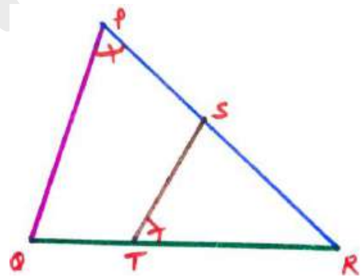
$\frac{QR}{QS} = \frac{QT}{QP}$ [From (2)]

$\angle Q = \angle Q$ [Common]

$\therefore \triangle PQS \sim \triangle TQR$ [SAS similarity]

5. S and T are points on sides PR and QR of $\triangle PQR$ such that $\angle P = \angle RTS$. Show that $\triangle RPQ \sim \triangle RTS$.

Sol. Given: In $\triangle$ PQR, $\angle P = \angle RTS$



To prove: $\triangle$ RPQ ~ $\triangle$ RTS

Proof: In $\triangle RPQ$ and $\triangle RST$,

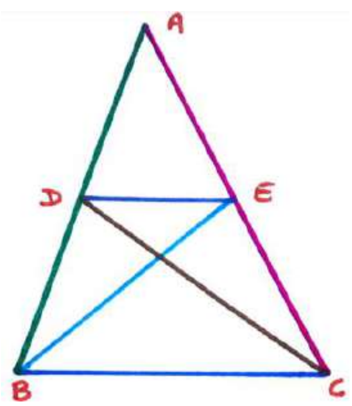
$\angle RTS = \angle QPS$ [Given]

$\angle R = \angle R$ [Common]

$\therefore \triangle RPQ \sim \triangle RTS$ [AA similarity]

6. In Figure, if $\triangle ABE \cong \triangle ACD$, show that $\triangle$ ADE ~ $\triangle$ ABC.

Sol. Given: $\triangle ABE \cong \triangle ACD$.



To prove: $\triangle$ ADE ~ $\triangle$ ABC

Proof: Since, $\triangle ABE \cong \triangle ACD$.

$\therefore AE = AD$ [c.p.c.t.]

Or, $AD = AE$.............. (i)

and, $AB = AC$ [c.p.c.t.] ....(ii)

$\therefore$ In $\triangle$ ADE and $\triangle$ ABC

$\frac{AD}{AB} = \frac{AE}{AC}$ [divide (i) by (ii)]

and $\angle A = \angle A$ [Common]

$\therefore$ $\triangle$ ADE ~ $\triangle$ ABC [SAS similarity]

7. In Figure, altitudes AD and CE of $\triangle ABC$ intersect each other at the point P. Show that:

(i) $\triangle AEP \sim \triangle CDP$ (ii) $\triangle ABD \sim \triangle CBE$

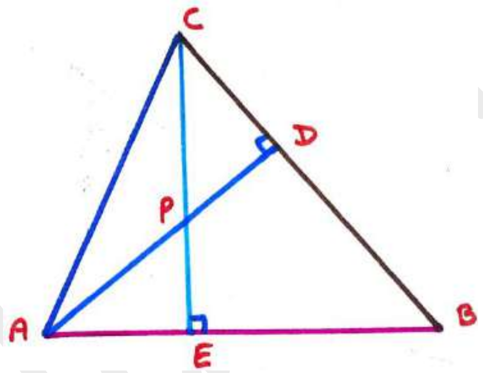
(iii) $\triangle AEP \sim \triangle ADB$ (iv) $\triangle PDC \sim \triangle BEC$

Sol. (i) In $\triangle$ AEP and $\triangle$ CDP,

$\angle APE = \angle CPD$ [Vertically Opposite Angles]

$\angle AEP = \angle CDP$ [Each $90°$]

$\therefore \triangle AEP \sim \triangle CDP$ [AA similarity]



(ii) In $\triangle$ ABD and $\triangle$ CBE,

$\angle ADB = \angle CEB$ [Each $90°$]

$\angle ABD = \angle CBE$ [Common]

$\therefore \triangle ABD \sim \triangle CBE$ [AA similarity]

(iii) In $\triangle$ AEP and $\triangle$ ADB,

$\angle AEP = \angle ADB$ [Each $90°$]

$\angle PAE = \angle DAB$ [Common]

$\therefore \triangle AEP \sim \triangle ADB$ [AA similarity]

(iv) In $\triangle$ PDC and $\triangle$ BEC,

$\angle PDC = \angle BEC$ [Each $90°$]

$\angle PCD = \angle BCE$ [Common]

$\therefore \triangle PDC \sim \triangle BEC$ [AA similarity]

8. E is a point on the side AD produced of a parallelogram $ABCD$ and BE intersects CD at F. Show that $\triangle ABE \sim \triangle CFB$.

Sol. Given: In llgm $ABCD$, E is a point on the side AD produced and BE intersects CD at F.

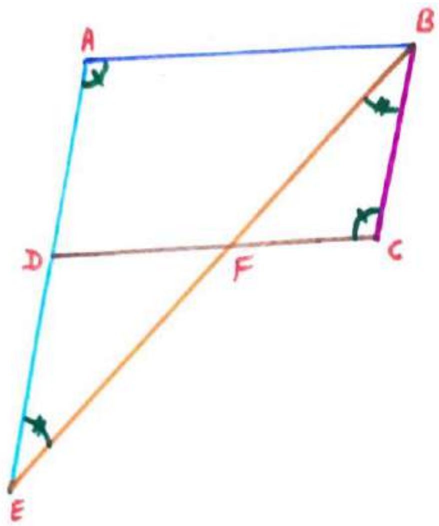
To prove: $\triangle ABE \sim \triangle CFB$

Proof: In $\triangle ABE$ and $\triangle CFB$,

$\angle A = \angle C$ [Opposite angles of parallelogram]

$\angle AEB = \angle CBF$ [Alternate angles as AE $\parallel BC$]

$\therefore \triangle ABE \sim \triangle CFB$ [AA similarity]



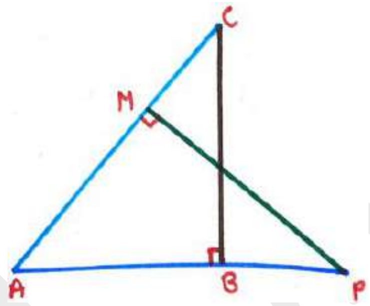
9. In Figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that

(i) $\triangle ABC \sim \triangle AMP$ (ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Sol. Given: In $\triangle ABC$ and $\triangle AMP$ angle B and M are right angle.

To prove: (i) $\triangle ABC \sim \triangle AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$



Proof: (i) In $\triangle ABC$ and $\triangle AMP$,

$\angle ABC = \angle AMP$ [Each $90°$]

$\angle A = \angle A$ [Common]

$\therefore \triangle ABC \sim \triangle AMP$ [AA similarity]

$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$ [$\because$ Corresponding parts of similar triangles are proportional]

10. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of $\triangle ABC$ and $\triangle EFG$ respectively. If $\triangle ABC \sim \triangle FEG$, show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$ (ii) $\triangle DCB \sim \triangle HGE$ (iii) $\triangle DCA \sim \triangle HGF$

Sol. Given: $\triangle ABC$ and $\triangle EFG$, CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$.

To prove: (i) $\frac{CD}{GH} = \frac{AC}{FG}$ (ii) $\triangle DCB \sim \triangle HGE$ (iii) $\triangle DCA \sim \triangle HGF$

Proof: (i) Since $\triangle ABC \sim \triangle FEG$

$\therefore \angle A = \angle F, \angle B = \angle E$

and $\angle ACB = \angle FGE, \angle ACB = \angle FGE$

$\therefore \angle ACD = \angle FGH$ [CD and GH are the bisectors of equal angles]

and, $\angle DCB = \angle HGE$ [CD and GH are the bisectors of equal angles]

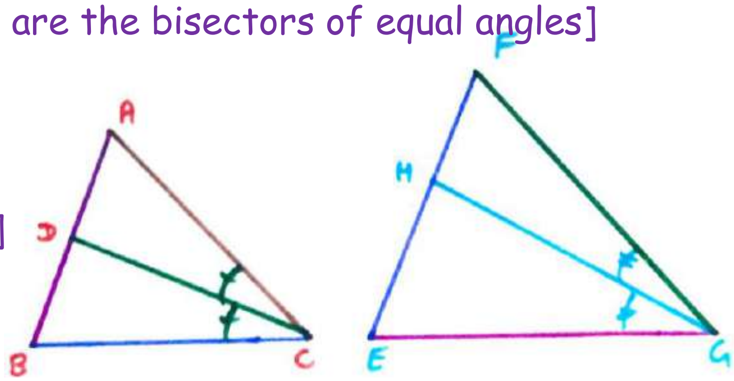
In $\triangle ACD$ and $\triangle FGH$,

$\angle A = \angle F$, and

$\angle ACD = \angle FGH$ [Proved above]

$\therefore \triangle ACD \sim \triangle FGH$ [AA similarity]

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$



(ii) In $\triangle DCB$ and $\triangle HGE$,

$\angle DCB = \angle HGE$ [Proved above]

$\angle B = \angle E$ [Proved above]

$\therefore \triangle DCB \sim \triangle HGE$ [AA similarity]

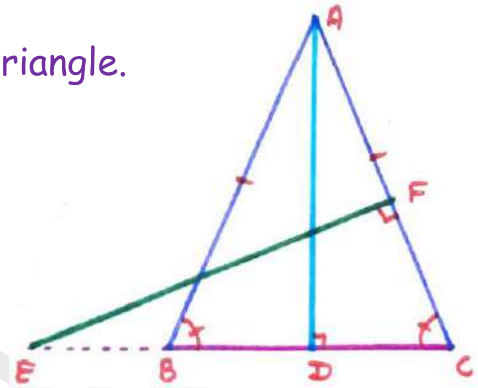
(iii) In $\triangle DCA$ and $\triangle HGF$,
$\angle ACD = \angle FGH$ [Proved above]
$\angle A = \angle F$ [Proved above]
$\therefore \triangle DCA \sim \triangle HGF$ [AA similarity]

11. In Figure, E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$.

Sol. Given: In $\triangle ABC$, $AB = AC$ and $AD \perp BC$ and $EF \perp AC$.

To prove: $\triangle ABD \sim \triangle ECF$.

Proof: Since $\triangle ABC$ is an isosceles triangle.
$\Rightarrow AB = AC$
$\Rightarrow \angle ABD = \angle ECF$
In $\triangle ABD$ and $\triangle ECF$,
$\angle ADB = \angle EFC$ [$\because$ Each $90°$]
$\angle ABD = \angle ECF$ [Proved above]
$\therefore \triangle ABD \sim \triangle ECF$ [AA similarity]



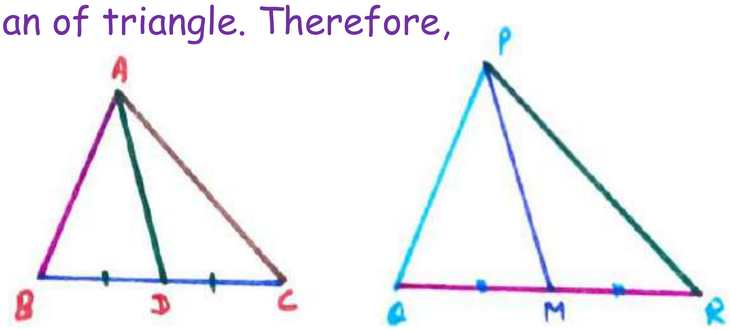
12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$ (see Figure). Show that $\triangle ABC \sim \triangle PQR$.

Sol. Given: In $\triangle ABC$ sides AB and BC and median AD are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$.

To prove: $\triangle ABC \sim \triangle PQR$.

Proof: AD and PM are the median of triangle. Therefore,

$BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$



Given that, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

In $\triangle ABD$ and $\triangle PQM$,
$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$ [Proved above]
$\therefore \triangle ABD \sim \triangle PQM$ [SSS similarity]

$\Rightarrow \angle ABD = \angle PQM$ [Corresponding angles of similar triangles]

In $\triangle$ ABC and $\triangle$ PQR,

$\angle ABD = \angle PQM$ [Proved above]

$\frac{AB}{PQ} = \frac{BC}{QR}$

$\therefore \triangle ABC \sim \triangle PQR$ [SAS similarity]

13. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB.CD$.

Sol. Given: In $\triangle ABC$, $\angle ADC = \angle BAC$

To prove: $CA^2 = CB.CD$.

Proof: In $\triangle$ ADC and $\triangle$ BAC,

$\angle ADC = \angle BAC$ [Given]

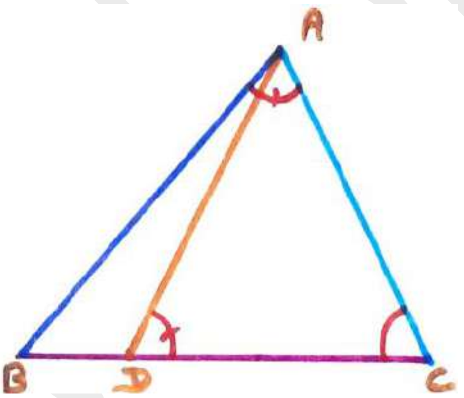
$\angle ACD = \angle BCA$ [[Given]

$\therefore \triangle ADC \sim \triangle BAC$ [AA similarity]

We know that the corresponding sides of similar triangles are proportional.

Therefore, $\frac{CA}{CB} = \frac{CD}{CA}$

$\Rightarrow CA^2 = CB \times CD$



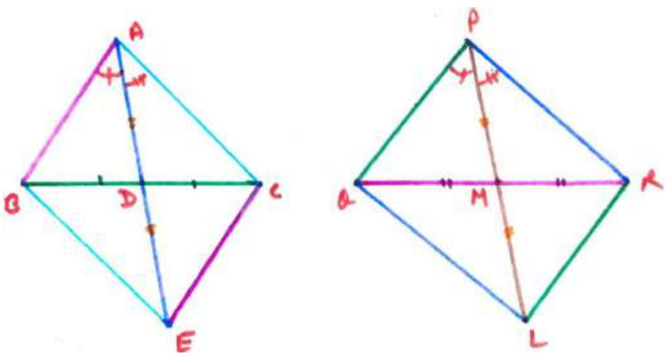
14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\triangle ABC \sim \triangle PQR$.

Sol. Given: In $\triangle$ ABC sides AB and AC and median AD are respectively proportional to sides PQ and PR and median PM of $\triangle$ PQR.

i.e., $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

To prove: $\triangle ABC \sim \triangle PQR$.

Construction: Produce AD and PM to E and L such that AD = DE And PM = DE. Now join BE, CE, QL and RL.



Proof: In quadrilateral ABEC, D is the mid-point of diagonal AE and BC, it implies that the diagonals bisect each other at D.

⇒ ABEC is a llgm.

Similarly, PQLR is also a llgm.

$\therefore$ BC = BE, AB = EC and PR = QL, PQ = LR [opposite sides of llgm]

$\Rightarrow \frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$

$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$

$\therefore \triangle ABE \sim \triangle PQL$

$\Rightarrow \frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

$\Rightarrow \frac{EC}{QL} = \frac{AC}{PR} = \frac{2AD}{2PM}$

$\Rightarrow \frac{EC}{QL} = \frac{AC}{PR} = \frac{AE}{PL}$

$\therefore \triangle ACE \sim \triangle PRL$ [SSS similarity]

We know that the corresponding angles of similar triangles are equal. efore

$\therefore \angle BAE = \angle QPL$ and $\angle CAE = \angle RPL$

Adding these two

$\angle BAE + \angle CAE = \angle QPL + \angle RPL$

$\Rightarrow \angle BAC = \angle QPR$

Now, in $\triangle ABC$ and $\triangle PQR$,

$\frac{AB}{PQ} = \frac{AC}{PR}$ [given]

And $\angle BAC = \angle QPR$ [proved above]

$\therefore \triangle ABC \sim \triangle PQR$ [SAS similarity] Proved.

15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Sol. Let AB be the pole and DE be the tower.

$\therefore$ BC and EF are the shadows of pole and tower.

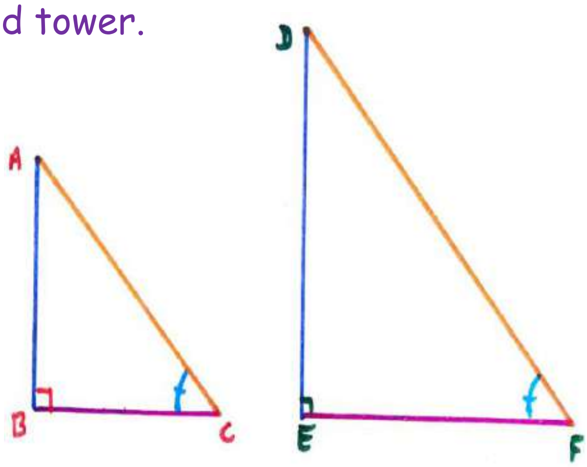
In $\triangle ABC$ and $\triangle DEF$,

$\angle ACB = \angle DFE$

[Angle of sun at the same time]

$\angle ABC = \angle DEF$ [Each 90°]

$\therefore \triangle ABC \sim \triangle DEF$ [AA similarity]



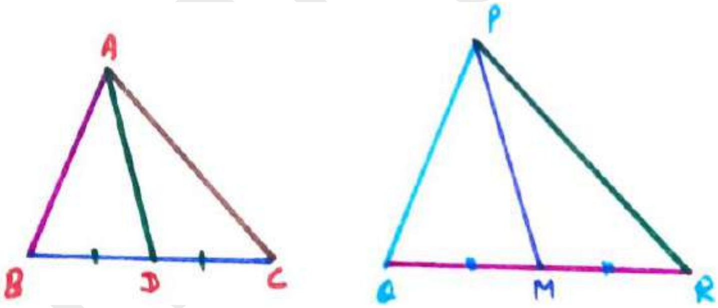
$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF}$ [∵ corresponding parts of similar triangles are proportional]
$\Rightarrow \frac{6}{DE} = \frac{4}{28}$
$\Rightarrow 6 \times 28 = DE \times 4$
$\Rightarrow AB = 42$ m

∴ The height of the tower is 42m. Ans.

16. If AD and PM are medians of triangles ABC and PQR, respectively where $\triangle ABC \sim \triangle PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Sol. Given: AD and PM are medians of triangles ABC and PQR

To prove: $\frac{AB}{PQ} = \frac{AD}{PM}$



Prove: Since $\triangle ABC \sim \triangle PQR$

We know that the corresponding sides of similar triangles are proportional. Therefore

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ ....(i)

and, $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$..........................(ii)

AD and PM are medians of triangle. Therefore

$BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$ ....(iii)

From the equation (i) and (iii), we have

$\frac{AB}{PQ} = \frac{BD}{QM}$..............................(iv)

In $\triangle ABD$ and $\triangle PQM$,

$\angle B = \angle Q$ [From the equation (ii)]

$\frac{AB}{PQ} = \frac{BD}{QM}$ [From the equation (iv)]

∴ $\triangle ABD \sim \triangle PQM$ [SAS similarity]

∴ $\frac{AB}{PQ} = \frac{AD}{PM}$. Proved.